

Symmetry breaking in nonequilibrium systems: Interaction of defects

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The dynamics of defects against the background of a regular system of rolls is investigated within the Swift-Hohenberg model with periodically-space-modulated pumping. For thermal convection, this corresponds to periodic nonuniform heating. Amplitude equations obtained for a parametric-resonance case generalize the equations derived earlier and describe both longitudinal and transverse defects. It is shown that a pair of longitudinal defects either attract one another until they annihilate or they diverge to infinity. Transverse defects are able to form bound static states. We investigate the influence of small nongradient effects on the dynamics of single longitudinal and transverse defects as well as on their interaction. We also discuss the effect of "elastic" scattering of transverse defects on each other in the presence of small dispersion.

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I. INTRODUCTION

Defects in regular spatiotemporal structures are a common phenomenon both in experiment and in numerical simulation. Defects can arise from spontaneous symmetry breaking due to secondary instabilities [1,2] or can be induced by external fields [3,4] or boundary conditions [5]. Although many mechanisms are available for the production of defects, only a few qualitatively different types are created. Therefore it is reasonable to study the dynamics and interaction of defects within simple models as the Ginzburg-Landau or the Swift-Hohenberg equations which allow for a sufficiently complete analytical investigation, on the one hand, and on the other hand, are general enough to be compared with available experimental data.

We will concern ourselves with the breaking of symmetry of field structures in the form of rolls that are formed in two-dimensional nonequilibrium media in a supercritical case. These structures include, for example, a lattice of rolls in Rayleigh-Bénard convection in a horizontal layer of fluid [6,7], a system of rolls in liquid crystals under electrohydrodynamic or convective instability [8,11], etc. The simplest among defects are one-dimensional domain walls. There are two qualitatively different types of walls in a roll system (Fig. 1). Figure 1(a) sketches a one-dimensional defect whose generating line is parallel to the axis of rolls. The breaking of symmetry is manifested, in this particular case, by the so-called "phase slip" in the core of the defect. Another one-dimensional defect (transverse shift) is shown in Fig. 1(b). Here, again, the plane is separated into two parts but now across rather than along the axis line of rolls. The rolls having the same direction of rotation in the different halves of the plane are shifted with respect to each other by a half-period of the lattice.

One-dimensional defects in nonequilibrium media are usually unstable and tend either to spread or to form two-dimensional topological defects—spirals or vortices [12]. However, they can be stable if additional space or time modulation breaks the continuous translational sym-

metry of the system. Much attention has been given to the evolution of nonequilibrium systems with spatially modulated forcing (e.g., [3,8–10]). If the spatial periods of modulation and of rolls in an unmodulated system are commensurate, forcing selects the orientation of rising rolls and suppresses their possible two-dimensional instabilities. If, however, the forcing is nonresonant, a kind of commensurate-noncommensurate transition occurs that gives rise to a number of one-dimensional defects [8,9]. Defects, however, are possible even in the case of exact parametric resonance when the wave number of modula-

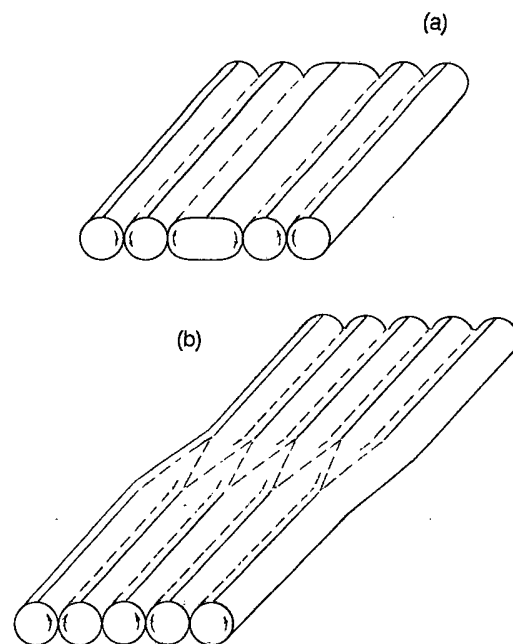


FIG. 1. Sketches of one-dimensional defects in a roll system. (a) *Longitudinal* defect, with a generating line parallel to the axes of rolls. In this particular case it has a form of the "phase slip" in the core of the defect; (b) *transverse* defect, with a generating line perpendicular to the axes of rolls. The phases of rolls in the opposite sides of the core of the defect differ by π .

tion is equal to twice the wave number of rolls. These defects asymptotically connect two regions where regular systems of rolls, both synchronized by a spatial forcing, are shifted in phase by an odd multiple of π (in fact, of the period of modulation). Analogous defects exist also in case of parametric temporal modulation of supercriticality in a one-dimensional system which undergoes a spatially homogeneous Hopf bifurcation [13]. It is recognized now that defects play an important role in the transition from a regular to a disordered state in a spatially extended system. Near the onset of the transition the interaction of defects seems to be responsible for the complex spatiotemporal behavior which is usually called defect-mediated turbulence.

In this paper we will investigate pair interactions of longitudinal and transverse defects in a system of rolls in the presence of parametric spatial forcing. We include the case of weak nongradient effects as dispersion, linear, and/or nonlinear frequency shift, which usually spoil the pure gradient dynamics of nonequilibrium systems. We will show that when two longitudinal defects interact, they either repel or annihilate as $t \rightarrow \infty$, resulting in the onset of a spatially homogeneous regime. The interaction of transverse defects is accompanied by a number of nontrivial effects, such as the formation of bound states as well as the regeneration and "elastic" scattering of defects in a nongradient case.

II. MODEL EQUATIONS

Our starting point in the description of both longitudinal and transverse defects in a roll lattice is the Swift-Hohenberg equation [14] for some field variable $\psi(\mathbf{r}, t)$, where $\mathbf{r}=(x, y)$, which reads (in nondimensional variables) as follows:

$$\frac{\partial \psi}{\partial t} = \bar{\mu} \psi - \psi^3 + (1 + \nabla^2)^2 \psi. \quad (1)$$

Here $\bar{\mu}(\mathbf{r})$ is a supercriticality which is a specified function of space, and the wave number of the most linearly unstable perturbations is normalized to 1. When $\bar{\mu} \geq 0$, this model describes, in particular, convection in a layer of fluid with rigid boundaries that is heated from below. In the following we will take $\bar{\mu} = \mu(1 + \varepsilon \cos kx)$, $\mu = \text{const}$, which corresponds to convection with spatially periodic heating. Below we will restrict ourselves to the case of exact parametric resonance $k=2$.

Equation (1) can be rewritten in gradient form,

$$\frac{\partial \psi}{\partial t} = - \frac{\delta F}{\delta \psi}, \quad (2)$$

where F stands for the free-energy functional

$$F = \int \int_{-\infty}^{\infty} \left\{ -\frac{1}{2} \bar{\mu} \psi^2 + \frac{1}{4} \psi^4 + \frac{1}{2} [(1 + \nabla^2) \psi]^2 \right\} dx dy. \quad (3)$$

We are interested in solutions $\psi(\mathbf{r}, t)$ of (2) with a definite symmetry, namely, $\psi(\mathbf{r}, t) = A(\mathbf{r}, t)e^{i\mathbf{x}} + \text{c.c.}$ Here $A(\mathbf{r}, t)$ is a slowly varying complex amplitude of the field. From Eq. (2) one can show that the equation for $A(\mathbf{r}, t)$ also has a gradient form:

$$\frac{\partial A}{\partial t} = - \frac{\delta \mathcal{F}}{\delta A^*}. \quad (4)$$

The free-energy functional \mathcal{F} for the amplitude A may be obtained directly by averaging F over the period of a lattice of rolls 2π (this approach is actually the generalization of Whitham's averaged variational principle [15] to gradient systems):

$$\mathcal{F} = \int \int_{-\infty}^{\infty} \left\{ -\mu |A|^2 + \frac{1}{2} |A|^4 - \gamma [(\text{Re } A)^2 - (\text{Im } A)^2] + \left[\left[\frac{\partial}{\partial x} - \frac{i}{2} \frac{\partial^2}{\partial y^2} \right] A \right]^2 \right\} dx dy, \quad (5)$$

where $\gamma = \frac{1}{2} \mu \varepsilon$. The evolution equation for $A(\mathbf{r}, t)$ can be written as a generalized Newell-Whitehead-Segel [16] equation:

$$\frac{\partial A}{\partial t} = \mu A + \gamma A^* - |A|^2 A + \left[\frac{\partial}{\partial x} - \frac{i}{2} \frac{\partial^2}{\partial y^2} \right]^2 A, \quad (6)$$

where the additional term γA^* is responsible for the parametric forcing.

Below we will consider two particular cases: purely longitudinal effects [with $\partial/\partial y = 0$ in Eq. (6)] and purely transverse defects (with $\partial/\partial x = 0$).

III. LONGITUDINAL DEFECTS

The dynamics of longitudinal defects [see Fig. 1(b)] is described by the following equation:

$$\frac{\partial A}{\partial t} = \mu A + \gamma A^* - |A|^2 A + \frac{\partial^2 A}{\partial x^2}. \quad (7)$$

This equation and the structure of a single longitudinal defect were discussed in the recent paper [13] where the authors analyzed the spatiotemporal dynamics of a distributed self-oscillatory system in a homogeneous external field oscillating with a frequency $2\omega_0$ (where ω_0 is the natural frequency of the system). In addition, Eq. (7) was considered earlier in [17,18], where it was shown that Eq. (7) describes two types of domain walls. When $\gamma < \frac{1}{3}$, the stable solution is

$$A = \pm \sqrt{\mu + \gamma} \tanh(\sqrt{2\gamma} x) \pm i \sqrt{\mu - 3\gamma} [(\sqrt{2\gamma} x)]^{-1}, \quad (8)$$

known as a Bloch wall, while for $\gamma > \frac{1}{3}$ this solution loses stability and, instead, the Ising domain wall

$$A = \pm \sqrt{\mu + \gamma} \tanh[\sqrt{\frac{1}{2}(\mu + \gamma)} x] \quad (9)$$

becomes stable. The solution (8) has an imaginary part A . This means that in (8) the phase changes smoothly from π to $-\pi$. In (9) the phase at the point $x=0$ (the defect core) changes abruptly.

We will now consider the interaction of a pair of longitudinal defects. If the defects are rather far apart in comparison with their characteristic scale, we can use an asymptotic method to derive equations for the coordinates of the defect core. The procedure for the derivation of equations describing the evolution of defect's parameters in the weak field of another defect is similar to the corresponding asymptotic procedure for solitons [19]. After some calculations (see the Appendix) we arrive at the equations for the distance r between two defects. In

particular, for two Bloch walls we have

$$\frac{dr}{dt} = \pm \frac{24(\mu - 3\gamma)}{3\mu - \gamma} \sqrt{2\gamma} e^{-\sqrt{2\gamma}r}. \quad (10)$$

The plus or minus sign in Eq. (10) is chosen depending on the ratio of the signs of the imaginary parts of complex amplitudes for the fields of interacting walls. Defects having $\text{Im} A$ equal in sign ("like" defects) attract one another (the minus sign), while "unlike" defects repel (the plus sign). For Ising walls we have

$$\frac{dr}{dt} = 12\sqrt{2(\mu + \gamma)} e^{-\sqrt{2(\mu + \gamma)}r}. \quad (11)$$

Let us compare the results obtained using an asymptotic method and the data of direct numerical simulation of the dynamics of defects. Equation (7) with periodic boundary conditions was integrated by the implicit split-step method using a fast Fourier transform (FFT) for the calculation of spatial derivatives (see [20]). We used 1024

harmonics, and the length of the system was $L=32$. Periodic boundary conditions $A(x)=A(x+L)$ were used in the numerical simulations. This corresponds to a periodic lattice of defects in the unbounded system. Therefore, for correct comparison with the asymptotic formulas (10) and (11), two terms describing interaction with two nearest neighbors should be taken into account in their right-hand sides. In Fig. 2(a) the velocity of convergence of "like" Bloch walls is plotted versus the distance between the walls for $\mu=1.0$, $\gamma=0.3$. For comparison, the solid line in this figure shows the velocity calculated analytically using the formula

$$v = \frac{24(\mu - 3\gamma)}{3\mu - \gamma} \sqrt{2\gamma} (e^{-\sqrt{2\gamma}r} - e^{-\sqrt{2\gamma}(L-r)}), \quad (12)$$

which follows from Eq. (10). One can see that the agreement is rather good for distances $r \gtrsim 8 \approx 3\sqrt{2}/\gamma$, where $\sqrt{2}/\gamma$ is the characteristic width of the Bloch wall. Accordingly, Fig. 2(b) presents numerical data for the repelling velocity of Ising walls for $\mu=1.0$, $\gamma=0.5$, as compared with the asymptotic formula (11).

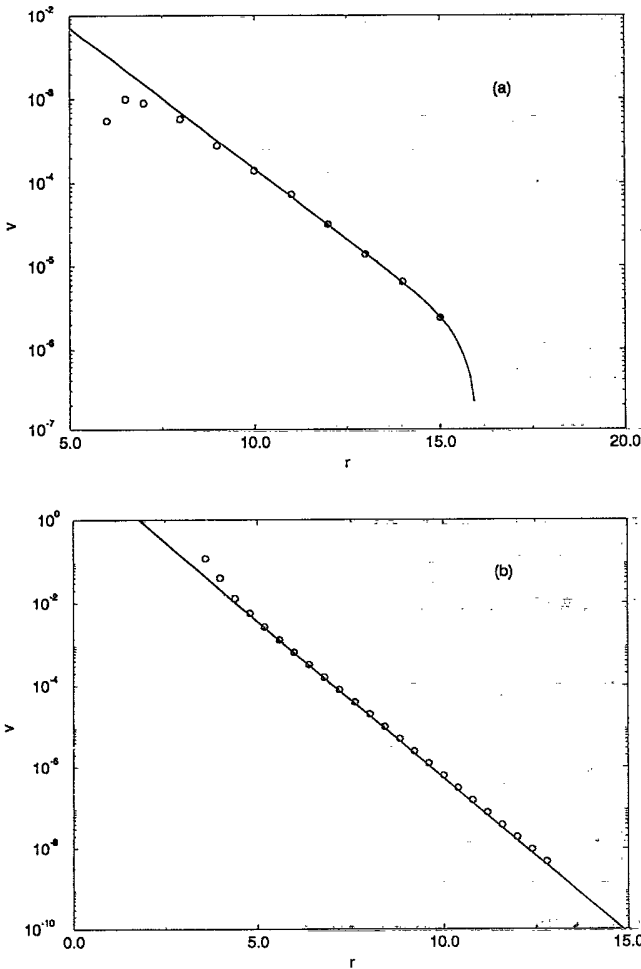


FIG. 2. (a) Convergence velocity of two longitudinal "like" Bloch walls as a function of their separation, as given by the analytical formula (12) (solid line) and by numerical simulations of Eq. (7) (circles) at $\mu=1.0$, $\gamma=0.3$; (b) repelling velocity of two longitudinal Ising walls as a function of their separation, as given by the analytical formula (11) (solid line) and by numerical simulations (circles) at $\mu=1.0$, $\gamma=0.5$.

IV. TRANSVERSE DEFECTS

The dynamics of transverse defects is described by the equation

$$\frac{\partial A}{\partial t} = \mu A + \gamma A^* - |A|^2 A - \frac{\partial^4 A}{\partial y_1^4}, \quad (13)$$

where $y_1 = y\sqrt{2}$ (below, instead of y_1 we will write simply y). This equation, as Eq. (7), has stable stationary solutions which will be referred to, by analogy, as an Ising wall (which is stable when $\gamma > \gamma_c = 0.391\dots$) or as a Bloch wall (which is stable when $\gamma < \gamma_c$). For the bifurcation diagram see Fig. 3.

Stationary solutions of Eq. (13) are described by the ordinary differential equation that is obtained from Eq. (13)

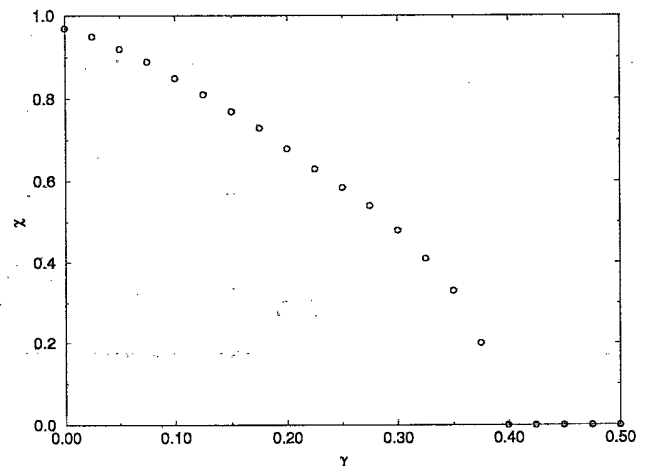


FIG. 3. Bifurcation diagram representing the parameter $\chi = \max\{\text{Im} A\}$ vs γ at $\mu=1.0$ for transverse defects. When $\gamma < \gamma_c = 0.392\dots$, Bloch walls are stable with $\chi \neq 0$. When $\gamma > \gamma_c$, Ising walls with $\text{Im} A \equiv 0$ are stable.

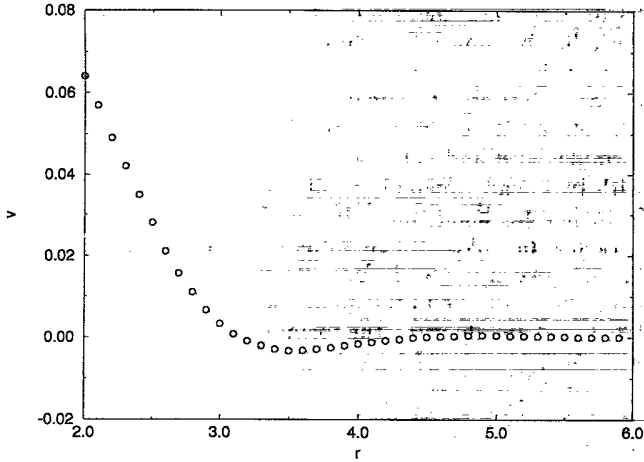


FIG. 4. Plot of the convergence velocity of two "unlike" transverse Bloch walls as a function of their separation, as given by numerical simulation of Eq. (13), $\mu=1.0$, $\gamma=0.3$.

when $\partial/\partial t \equiv 0$:

$$\mu A + \gamma A^* - A_{yyyy} - |A|^2 A = 0. \quad (14)$$

Equations (14) admits the "energy" integral

$$H = -A_y^* A_{yyy} - A_y A_{yyy}^* + |A_{yy}|^2 + \mu |A|^2 - \frac{1}{2} |A|^4 + \frac{\gamma}{2} (\text{Re} A)^2. \quad (15)$$

It can be shown that in the three-dimensional phase space of the system (14) and (15) there exists a homoclinic structure [1,21], which indicates that (14) is nonintegrable. However, the interaction of walls can be described knowing only the asymptotic forms of solutions (14) as $y \rightarrow \pm\infty$:

$$|A - A_0| \sim e^{(\pm\lambda \pm i\lambda)y}, \quad (16)$$

where $\lambda = \sqrt[3]{(\mu + \gamma)/2}$ if $\gamma > \gamma_c$ and $\lambda = \sqrt[3]{\gamma/2}$ if $\gamma < \gamma_c$; $A_0 = \pm \sqrt{\mu + \gamma}$.

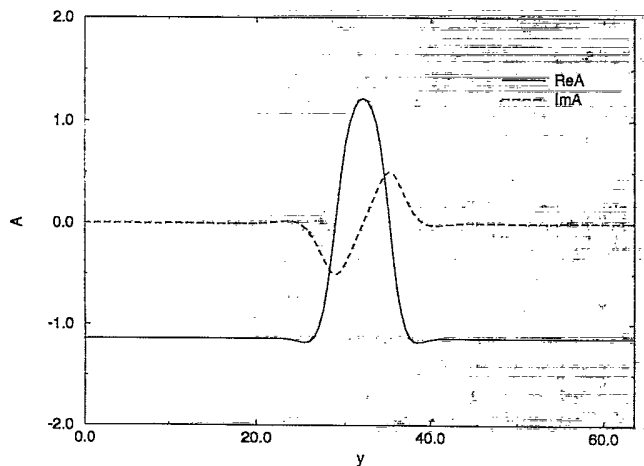


FIG. 5. Closest bound state of two "unlike" transverse Bloch walls, corresponding to the first zero of the convergence velocity as a function of r (see Fig. 4), $\mu=1.0$, $\gamma=0.3$.

Employing Eq. (A7) of the Appendix we again can derive an equation of motion for the distance between the cores of interacting transverse defects:

$$M \frac{dr}{dt} = C_0 e^{-\lambda r} \cos(\lambda r + \phi_0), \quad (17)$$

where the mobility of the defect $M = \int_{-\infty}^{\infty} |A_y'|^2 dy$ and C_0 and ϕ_0 are some constants which one can easily calculate numerically for any chosen values of parameters μ and γ . It is clear from Eq. (17) that there exists a countable number of stable equilibrium states with $r_n = \lambda^{-1}(2\pi n - \phi_0)$, $n=1, 2, \dots$. This conclusion is confirmed by numerical experiments in which we calculated the velocity of converging defects versus their relative distance (see Fig. 4). One of the family of bound states is depicted in Fig. 5.

V. NONGRADIENT TERMS IN THE GOVERNING EQUATIONS

It is typical for nonequilibrium systems that they reveal some nonvariational features such as oscillatory behavior of growing disturbances, dispersion (nonlinear frequency-wave-number dependence), nonlinear frequency shift, etc. These effects are described by imaginary corrections to the coefficients of the corresponding amplitude equations. In this section we consider the influence of nonvariational effects on the interaction of defects in the roll system.

A. Longitudinal defects

The motion of domain walls under the influence of small nonvariational terms in the governing equation

$$A_t = (\mu + iv)A + (1 + i\alpha)A_{xx} - (1 + i\beta)|A|^2 A + \gamma A^* \quad (18)$$

was investigated in Ref. [13]. It was found that small nongradient effects do not significantly influence the behavior of an Ising wall (it remains immobile), while a Bloch wall begins to move in a direction that depends on the direction of the phase turning in the defect (i.e., on the sign of $\text{Im} A$).

Two interacting Ising walls repel and move off to infinity regardless of nongradient terms. Bloch walls in the gradient model either attract each other or repel depending on the ratio of signs of $\text{Im} A$ in the defect cores. Under the action of nongradient effects, "like" defects move in one direction and, at the same time, attract each other until they annihilate completely, which results in the onset of a spatially homogeneous regime. "Unlike" defects move in different directions under the action of nongradient effects as well as due to interaction, and either annihilate or repel up to infinity. Thus, longitudinal defects do not form bound states under the action of weak nongradient effects.

B. Transverse defects

Let us first consider the influence of small nongradient effects on the dynamics of a single transverse defect.

These effects are described by the imaginary corrections to the coefficients of the amplitude equation (13):

$$A_t = (\mu + i\nu)A - (1 + i\alpha)A_{yyyy} - (1 + i\beta)|A|^2A + \gamma A^* \quad (19)$$

Following [13] and using the asymptotic method of series expansion in a small parameter $\varepsilon = O(\nu, \alpha, \beta)$ [Eq. (A7) again, but with different $h^{(1)}$], one can show that an Ising wall continues to be immobile while a Bloch wall moves at a constant velocity

$$v = \chi M^{-1}[(\beta - \nu)a_1 + (\alpha - \beta)a_2], \quad (20)$$

where

$$a_1 = \int_{-\infty}^{\infty} (Y_0 X'_{0y} - X_0 Y'_{0y}) dy \quad (21)$$

$$a_2 = \int_{-\infty}^{\infty} (X_{0yyyy} Y_{0y} - Y_{0yyyy} X_{0y}) dy, \quad (22)$$

$A_0 = X_0 + iY_0$ is the profile of an undisturbed transverse defect (when ν, α, β are zero), and $\chi = \max(\text{Im} A)$ in the core of undisturbed defect. In this case the number of bound states of the pair of interacting defects is finite, be-

cause the distance between defects in the two-defect steady bound states must meet the condition

$$C_0 e^{-\lambda r_n} \cos(\lambda r_n + \phi_0) = \chi[(\beta - \nu)a_1 + (\alpha - \beta)a_2], \quad (23)$$

and for large r this condition cannot be satisfied. Moreover, for sufficiently strong nongradient effects, when the right-hand side of Eq. (23) is larger than some threshold value,

$$|\chi[(\beta - \nu)a_1 + (\alpha - \beta)a_2]| > h_{\max},$$

there are no bound states at all. The threshold value h_{\max} is determined by the value of the first minimum (or maximum) on the oscillating tail $\text{Im} A$ of the transverse defect. "Like" Bloch walls moving in opposite directions to another usually annihilate, and a spatially homogeneous regime is established. However, when $\beta = 0$, $\nu = 0$, and $\alpha > \alpha_{\text{cr}} \cong h_{\max}/a_2$ there appears an interesting effect: regeneration of defects having different signs in the region of strong interaction. After interaction the defects diverge at a constant velocity because of the change of sign of $\text{Im} A$ in their cores. This process is very much like elastic scattering of particles (see Fig. 6), although actual-

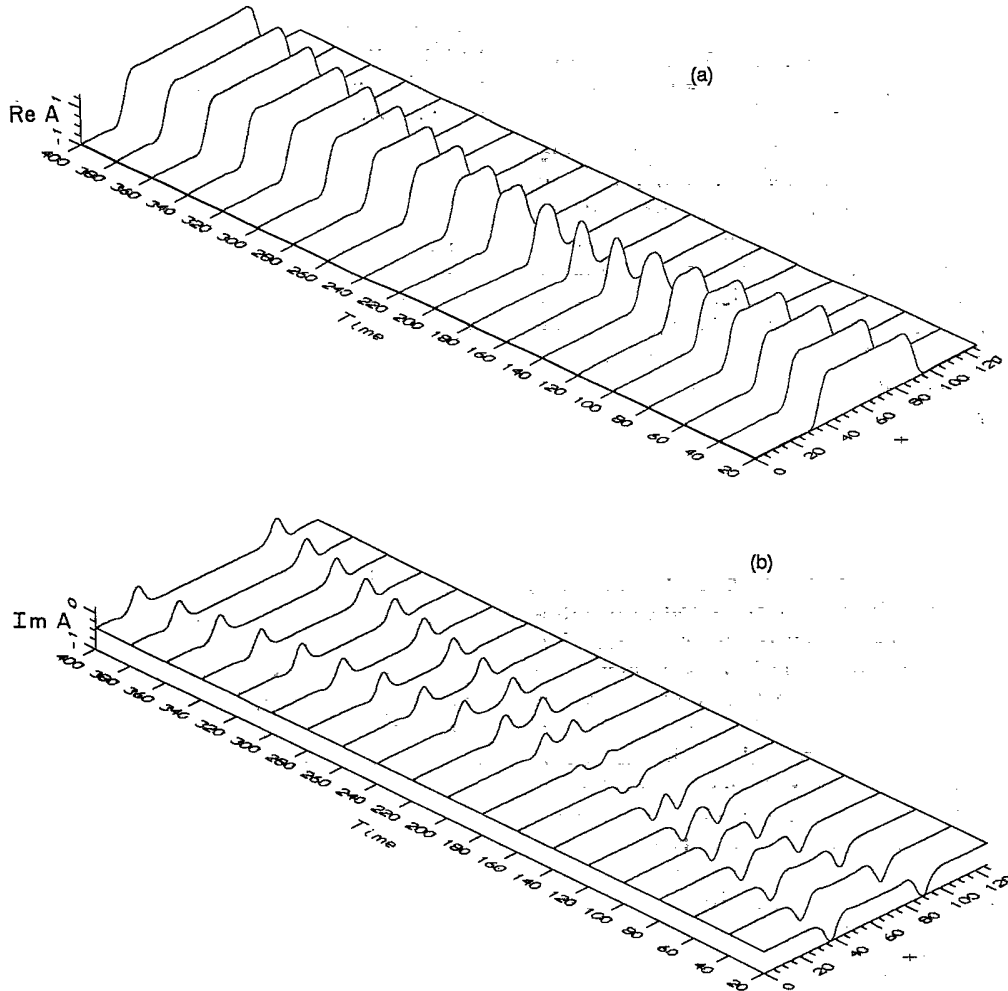


FIG. 6. Regeneration of "like" transverse Bloch walls under collision in the nongradient case ($\mu = 1.0$, $\gamma = 0.3$, $\alpha = 0.3$, $\beta = 0$, $\nu = 0$). (a) $\text{Re} A$, (b) $\text{Im} A$ as functions of y at different moments of time.

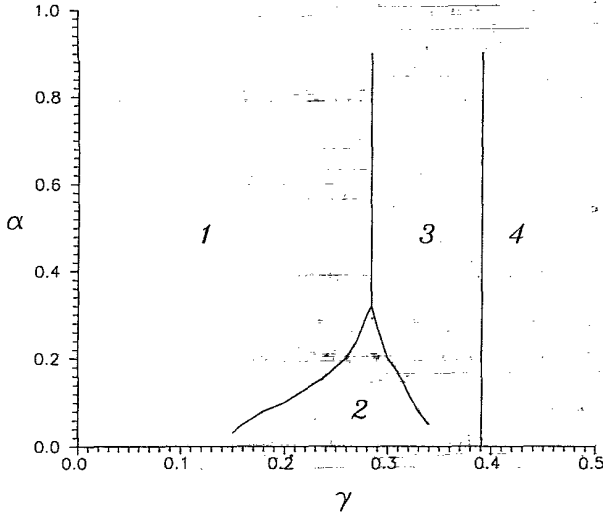


FIG. 7. Parameter plane (γ, α) for the pair interaction of transverse defects for $\beta = \nu = 0$. Domain 1 corresponds to the annihilation of Bloch walls, 2 to bound states of Bloch walls, 3 to the regeneration of Bloch walls, and 4 to bound states of Ising walls.

ly the interaction and propagation of defects is an essentially nonequilibrium process, accompanied by variations of the free-energy functional (5).

Figure 7 depicts the separation of the plane of parameters (γ, α) into regions that correspond to a qualitatively different dynamics of transverse defects (for simplicity we restrict ourselves to the case of $\nu = \beta = 0$). When $\gamma > \gamma_c = 0.391 \dots$, only Ising walls are stable; they begin to move under the action of nongradient effects. When $\gamma < \gamma_c$, we observe bound static states of defects (for not too large $\alpha < \alpha_c = 0.32 \dots$), annihilation of like defects for small γ , and regeneration of defects in the course of interaction at large γ .

VI. CONCLUSIONS

We have investigated one-dimensional defects that may occur in the system of field structures in the form of one-dimensional rolls under the action of periodically inhomogeneous pumping using a Swift-Hohenberg equation with a periodic-in-space supercriticality parameter. We considered the case of exact parametric resonance when the spatial period of rolls is twice the pumping period. Then, we averaged the equation over the period of pumping and arrived at a generalized Newell-Whitehead-Segel equation for the complex amplitude of the primary field. Topologically, two qualitatively different types of one-dimensional defects are possible in the system of parallel rolls: longitudinal defects (along the direction of rolls) and transverse defects. In a purely gradient case, defects are immobile and their internal structure depends on the magnitude of the external periodic inhomogeneity γ . For large γ , strictly antisymmetric Ising walls are formed, while for $\gamma < \gamma_c$, the symmetry is broken and Bloch walls become stable, with the phase of the complex amplitude

turning in different directions depending on initial conditions.

The qualitative difference between longitudinal and transverse defects is manifested in their interaction. Longitudinal defects either attract one another and annihilate ("like" Bloch walls) or diverge to infinity (Ising walls, "unlike" Bloch walls). On the other hand, two transverse defects form a bound state (in a purely gradient case there exists a countable set of bound states of two defects). Single Bloch walls begin to move, given there are imaginary corrections to the coefficients of equations for complex amplitude (these corrections may be related to dispersion, nonlinear frequency shift, or linear detuning). The number of bound states may decrease down to zero, depending on the value of nongradient terms, so that there are no bound localized states when nongradient effects are sufficiently large. In this case, two qualitatively different types of behavior of a pair of transverse defects are possible: (1) the defects diverge to infinity or converge and annihilate, giving rise to a spatially homogeneous regime, or (2) defects are regenerated with the opposite direction of the phase rotation, and the newly born defects diverge. In the region of parameters corresponding to the interaction of the second type, a system of a large number of transverse defects would demonstrate complex nonstationary behavior that may be interpreted as one-dimensional defect-mediated turbulence.

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APPENDIX

Let us seek the solutions to Eq. (7) in the form

$$A = A_1[x - r(t)/2] + A_2[x + r(t)/2] - \sqrt{\mu + \gamma} + \sum_n \epsilon^n a^{(n)}(x, t), \quad (\text{A1})$$

where A_1 and A_2 stand for undisturbed domain walls (8) or (9) taken with the opposite signs. $\sqrt{\mu + \gamma}$ is added to meet the boundary conditions $A \rightarrow \sqrt{\mu + \gamma}$ at $x \rightarrow \pm \infty$, and the small parameter $\epsilon = (\sqrt{2\gamma r})^{-1}$ is the ratio of the defect width and a separation of defects r which is a function of "slow" time $\tau = \epsilon t$. The expansion (A1) substituted into Eq. (7) yields in a standard manner the set of equations for successive approximations $a^{(n)}$:

$$\hat{L}_{1,2} \begin{pmatrix} a^{(n)} \\ a^{*(n)} \end{pmatrix} = \hat{H}_{1,2}^{(n)}, \quad (\text{A2})$$

where, for instance,

$$\hat{L}_1 = \begin{pmatrix} \frac{\partial}{\partial t} - \mu - \partial^2/\partial x^2 + 2|A_1|^2 & -\gamma + A_1^2 \\ -\gamma + (A_1^*)^2 & \frac{\partial}{\partial t} - \mu - \partial^2/\partial x^2 + 2|A_1|^2 \end{pmatrix} \quad (\text{A3})$$

is the operator of the problem (7) linearized with respect to the first defect A_1 and $\hat{H}_{1,2}^{(n)}$ contain only the functions of previous corrections. The first-order correction $\hat{H}_1^{(1)}$ takes the form

$$\hat{H}_1^{(1)} = \epsilon^{-1} \begin{pmatrix} \frac{\partial A_1}{\partial t} + h^{(1)} \\ \frac{\partial A_1^*}{\partial t} + h^{(1)*} \end{pmatrix}, \quad (\text{A4})$$

$$h^{(1)} = -2|A_1|^2 \tilde{A}_2 - A_1^2 \tilde{A}_2^* - 2\sqrt{\mu + \gamma} (A_1 \tilde{A}_2 + A_1 \tilde{A}_2^* + A_1^* \tilde{A}_2), \quad (\text{A5})$$

where the tilde indicates that the first term in the asymptotic expansion of $A_2 - \sqrt{\mu + \gamma}$ is taken in the location of the first defect. The expressions for \hat{L}_2 and $\hat{H}_2^{(1)}$ may be obtained by the permutation of subscripts 1 and 2 in

(A3)–(A5). In order to avoid the secular growth of corrections $a^{(n)}$ one has to impose the orthogonality conditions between the right-hand side of (A2) and the eigenvectors $a_{1,2}^+$ of the operators conjugate to $\hat{L}_{1,2}$:

$$\int_{-\infty}^{\infty} a_{1,2}^+ \hat{H}_{1,2}^{(n)} dx = 0. \quad (\text{A6})$$

It is easy to see that due to the translation invariance of (7), the space derivatives $dA_{1,2}^*/dx$ and $dA_{1,2}/dx$ form the eigenvector of $\hat{L}_{1,2}$. Therefore, the orthogonality condition for first-order corrections may be represented in the following form:

$$\int_{-\infty}^{\infty} |A_{1x}|^2 dx \frac{dr}{dt} = \int_{-\infty}^{\infty} \frac{\partial A_1^*}{\partial x} h^{(1)} dx + \text{c.c.} \quad (\text{A7})$$

The substitution of (8) or (9) into (A7) and a straightforward calculation of integrals in (A7) yield the formulas (10) or (11), respectively.

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