Performance Analysis of Correlation-Based Communication Schemes Utilizing Chaos

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***Abstract*—Using chaotic signals in spread-spectrum communications has a few clear advantages over traditional approaches. Chaotic signals are nonperiodic, wide-band, and more difficult to predict, reconstruct, and characterize than periodic carriers. These properties of chaotic signals make it more difficult to intercept and decode the information modulated upon them. However, many suggested chaos-based communication schemes do not provide processing gain, a feature highly desirable in spread-spectrum communication schemes. In this paper, we suggest two communication schemes that provide a processing gain. The performance of these and of the earlier proposed differential chaos shift keying is studied analytically and numerically for discrete time implementations. It is shown that, when performance is characterized by the dependence of bit error rate on E_b/N_0, the increase of the spreading sequence length beyond a certain point degrades the performance. For a given E_b/N_0, there is a length of the spreading sequence that minimizes the bit error rate.**

I. Introduction

BROAD continuous spectra of chaotic signals make them very appealing for use as carriers in spread-spectrum communications [1]–[8]. Research in this direction has drawn considerable interest in the last few years. The accumulating volume of research has illuminated a few fundamental problems that researchers face in developing chaos-based communication schemes. One such problem is developing a chaos-based communication scheme that would achieve the so-called processing gain.

The idea behind spread-spectrum communication schemes [9], [10] is that a relatively narrow-band information signal is modulated upon a rather wide-band carrier. Common spread spectrum technologies utilize averaging or correlation techniques that match the received signal with a certain *a priori* known pattern. In these methods, the useful signal is accumulated coherently and the channel noise and interference are averaged out. The number of samples over which the averaging is done is usually called the length of the spreading sequence. As this number increases proportionally to the bandwidth of the carrier, the accumulated useful signal grows proportionally to it, while the interference grows at a slower rate. The property of spread-spectrum systems to suppress interference due to utilization of a wider bandwidth is called a processing gain. Some of the additional advantages of using spread-spectrum communications is that it is more resistant to narrow-band interference and has lower spectral power density, which makes it less detectable and less damaging to many existing narrow band communication systems.

Achieving processing gain is the key point in the design of spread-spectrum chaos-based communication systems. Clearly, this can only be done if the bandwidth of the chaotic carrier is wider than that of the information signal. This, however, is necessary, but is not a sufficient condition for providing a processing gain. For example, simply adding a wide-band chaotic signal to the information signal at the transmitter, and then subtracting it at the receiver, as in the chaotic masking scheme [11]–[14], does not produce any processing gain. Chaotic modulation technique in its original form [15], [16] does not yield any processing gain, either.

Incorporating processing gain into chaotic spread-spectrum systems is a nontrivial task because, on one hand, it is difficult to recreate the nonperiodic spreading sequence at the receiver operating in noisy environment, and on the other hand, designing a real-time matched filter to suppress noise in chaotic signals is just as difficult. This problem can be circumvented by actually transmitting both the reference signal and the information signal as in traditional *transmitted reference* (TR) methods. This approach is used in particular by the *differential chaos shift keying* (DCSK) [4], [17].

In this paper, we introduce two new correlation-based communication schemes: *correlation delay shift keying* (CDSK) and *symmetric chaos shift keying* (SCSK). In CDSK, instead of the sequential transmission of the modulated chaotic signal and the reference, as it is done in DCSK, the two are added together with a certain time delay. This permits a continuous operation of the transmitter and makes the transmitted signal more homogeneous and less prone to interception. SCSK, a subclass of antipodal chaos shift keying, uses a matched nonlinear system to reconstruct the reference signal, thus eliminating the need for transmitting it over the communication channel, simplifying the transmitter design, and providing some selectivity. Both proposed systems provide processing gain by employing correlation detection.

It was reported in [21] and [22] that DCSK differs substantially from the traditional *differential phase shift keying* (DPSK) and TR systems in that the carrier energy in DCSK can fluctuate chaotically. This problem can be avoided by chaotically modulating the frequency or the phase of high-frequency periodic carriers. The performance of such systems was extensively studied in [17], [18] and [20]. However, in some applications it may be desirable to spread the transmitted energy over as wide

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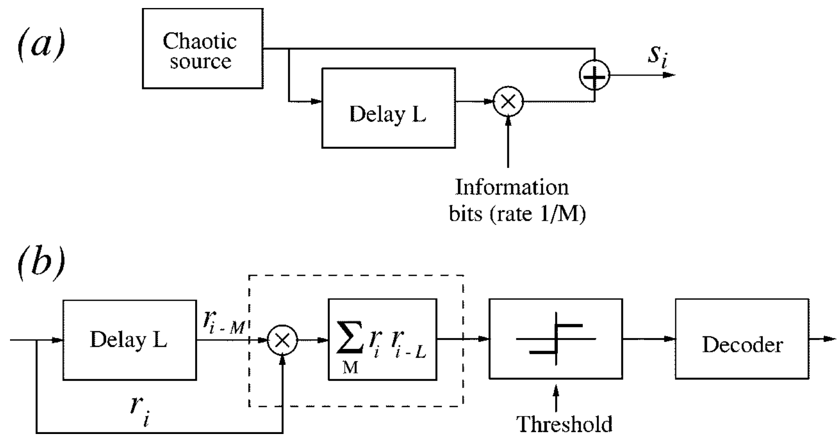

Fig. 1. DCSK operation: (a)—transmitter, (b)—receiver.

frequency range as possible. In these cases, it may be advantageous to use base-band implementations of DCSK and similar schemes. We study analytically and numerically the performance of base-band implementations of DCSK and the two new schemes, and show that both CDSK and SCSK perform 2–3 dB worse than DCSK. We also observe that each scheme shows the same characteristic behavior, namely, for every value of E_b/N_0, there is a certain length of the spreading sequence that minimizes the bit error rate. This behavior is explained in part by the variability of the bit energy for chaotic sequences [21], [22]. Despite their slightly inferior performance, CDSK and SCSK have certain attractive features that make them viable candidates for chaos-based spread spectrum communications.

II. System Description: DCSK, CDSK, SCSK

A. DCSK

The operation of the DCSK modulator and demodulator is illustrated in Fig. 1. For every bit of information, the transmitter outputs a chaotic sequence x_i of length M followed by the same sequence multiplied by the information signal $b_l = \pm 1$, where l is the bit counter. Thus, the transmitted signal s_i for a single bit is given by

$$s_i = \begin{cases} x_i, & 0 < i \le M \\ b_l x_{i-M}, & M < i \le 2M. \end{cases}$$

In order to recover the information, the received signal r_i is multiplied by the received signal delayed by M, r_{i-M}. The product is then averaged over the spreading sequence length M. Thus, the output of the correlator can be written as

$$S = \sum_{i=1}^{M} r_i r_{i+M}.$$

Let us make the standard assumptions that the received signal r_i is given by $r_i = s_i + \xi_i$, where ξ_i is a stationary random process with $\langle \xi_i \rangle = 0$, that ξ_i and ξ_j are statistically independent for any $i \neq j$, and that we can maintain perfect bit synchronization. Then the correlator output can be written as

$$\begin{aligned} S &= \sum_{i=1}^{M} (s_i + \xi_i)(s_{i+M} + \xi_{i+M}) \\ &= \sum_{i=1}^{M} (b_l x_i^2 + x_i(\xi_{i+M} + b_l \xi_i) + \xi_i \xi_{i+M}) \\ &= b_l \sum_{i=1}^{M} x_i^2 + \sum_{i=1}^{M} (x_i(\xi_{i+M} + b_l \xi_i) + \xi_i \xi_{i+M}) \end{aligned} \quad (1)$$

In this expression, the first term is the useful signal,[1] and the second is a zero-mean random quantity.

One shortcoming of this method, the need to transmit the same chaotic sequence twice, makes this system prone to interception. Also, the transmitter requires a delay element and a switch, or a generator capable of reproducing the same chaotic sequence. This can lead to technical implementation difficulties.

B. CDSK

In the CDSK modulator (Fig. 2), the transmitted signal is the sum of a chaotic sequence x_i and of the delayed chaotic sequence x_{i-L} multiplied by the information signal $b_l = \pm 1$: $s_i = x_i + b_l x_{i-L}$. Thus, CDSK overcomes the mentioned above disadvantages of DCSK: the switch in the transmitter is now replaced by an adder, and the transmitted signal is never repeated. It should be kept in mind, however, that a more sophisticated correlation analysis will still be able to detect a CDSK-based transmission. The receiver (Fig. 2(b)) is the same as for DCSK, except the delay L now does not have to be equal to the spreading sequence length M. The correlator output S is given by the sum

$$\begin{aligned} S &= \sum_{i=1}^{M} (x_i + b_l x_{i-L} + \xi_i)(x_{i-L} + b_{l-1} x_{i-2L} + \xi_{i-L}) \\ &= b_l \sum_{i=1}^{M} x_{i-L}^2 + \sum_{i=1}^{M} \eta_i \end{aligned} \quad (2)$$

[1]Strictly speaking, the first term in (1) is still a fluctuating quantity, due to the fact that the transmitted signal changes from bit to another bit. Thus, a part of this term should also be considered as contamination.

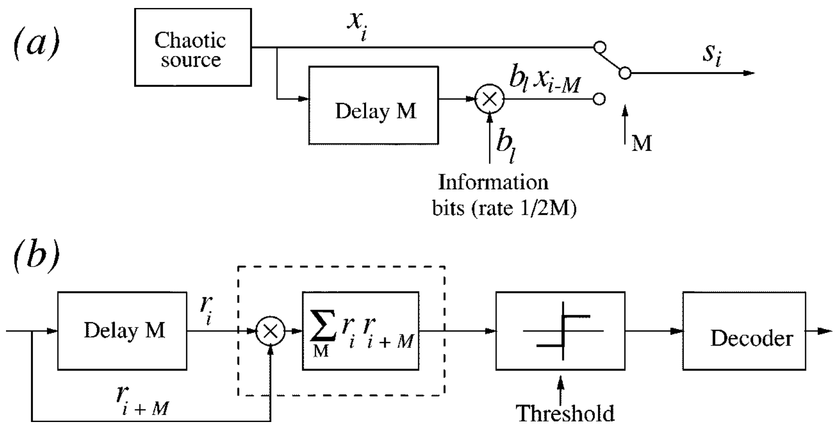

Fig. 2. CDSK operation: (a)—transmitter, (b)—receiver.

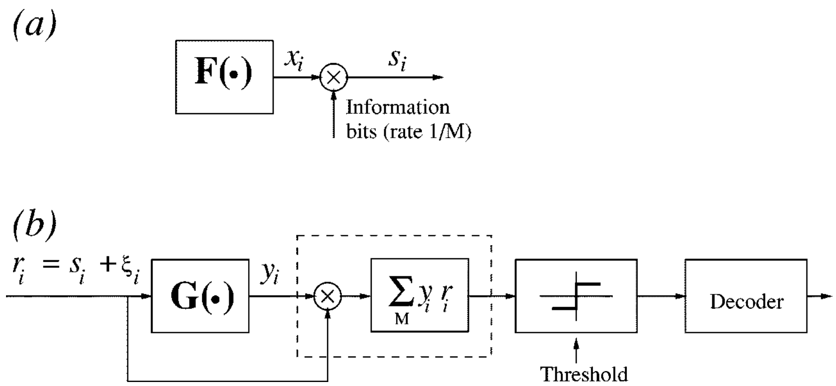

Fig. 3. SCSK operation: (a)—transmitter, (b)—receiver.

where

$$\begin{aligned}\eta_i =& x_i x_{i-L} + b_{l-1} x_i x_{i-2L} + b_l b_{l-1} x_{i-L} x_{i-2L}\\ &+ x_i \xi_{i-L} + b_l x_{i-L} \xi_{i-L} + x_{i-L} \xi_i\\ &+ b_{l-1} x_{i-2L} \xi_i + \xi_i \xi_{i-L}.\end{aligned}$$

The first term in (2) is the useful signal, and the second is the contamination which now comes not only from noise part of the correlator input, but also from correlating chaotic segments over finite time. As we shall see in the next section, this additional contribution leads to a somewhat inferior performance of CDSK, compared to DCSK.

C. SCSK

As an alternative to including the reference signal in the transmitted signal, the reference signal can be recreated in the receiver. This approach is taken in the design of SCSK, whose operation is illustrated in Fig. 3. The central element of a SCSK transmitter is the chaotic map

$$\mathbf{x}_{i+1} = \mathbf{F}(\mathbf{x_i})$$

where $\mathbf{x}_i$ is the internal state vector. The first component of this vector multiplied by the information signal $b_l = \pm 1$ is the transmitted signal: $s_i = b_l x_i$. In the receiver, this signal is driving a matched chaotic system:

$$\mathbf{y}_{i+1} = \mathbf{G}(|s_i|, \mathbf{y_i}).$$

We shall choose $\mathbf{F}(\bullet)$ and $\mathbf{G}(\bullet)$, such that the drive-response system that they form has a stable identically synchronous regime with respect to first components of vectors $\mathbf{x_i}$ and $\mathbf{y_i}$: $x_i^1 = y_i^1$. The simplest example of such drive-response system is two one-dimensional (1D) maps:

$$\begin{aligned}x_{i+1} =& F(x_i),\\ y_{i+1} =& F(s_i)\end{aligned}$$

where $F(\bullet)$ is even, $F(x) = F(-x)$. In a noise-free case, the output of the chaotic system in the receiver is the same as the output of the chaotic system in the transmitter, and the same as the signal in the channel, except for information-dependent polarity modulation. The sign of b_l can therefore be determined by taking the product of the received signal and the output of the chaotic system in the receiver. The product can then be averaged over the length of the spreading sequence, in order to reduce the effects of channel noise.

In general, the correlator output for SCSK can be written as

$$S = \sum_{i=1}^{M} y_i^1 (b_l x_i^1 + \xi_i)$$

where y_i^1 is the output of the chaotic system in the receiver. Let us only consider here the case when the chaotic map is one-dimensional. Then,

$$S = \sum_{i=1}^{M} F(b_l x_{i-1} + \xi_{i-1})(b_l x_i + \xi_i).$$

We can introduce $\tilde{\xi}_i = \xi_i / b_l$ and rewrite this in the form

$$\begin{aligned} S =& b_l \sum_{i=1}^{M} F\left(x_{i-1} + \tilde{\xi}_{i-1}\right)\left(F(x_{i-1}) + \tilde{\xi}_i\right) \\ =& b_l \sum_{i=1}^{M} x_i^2 + b_l \sum_{i=1}^{M} F\left(x_{i-1} + \tilde{\xi}_{i-1}\right)\tilde{\xi}_i \\ &+ b_l \sum_{i=1}^{M} \left(F\left(x_{i-1} + \tilde{\xi}_{i-1}\right) - F(x_{i-1})\right) F(x_{i-1}). \quad (3) \end{aligned}$$

The first sum in this expression is the useful signal, and the second is the interference.

SCSK approach has advantages over both DCSK and CDSK. The transmitter design is simpler. The SCSK transmitted sequence is nonrepeating, leading to lower probability of intercept. Additionally, demodulation of SCSK signal requires a matched nonlinear system in the receiver, thus offering better protection against an unauthorized reception. These advantages come at the expense of some performance loss and of the more restricted choice of nonlinear systems used for chaos generation.

III. Performance Analysis

A. DCSK Performance

The output of the correlator for DCSK is given by (1). It can be written in the form

$$S = b_l A + b_l \zeta + \eta, \qquad A > 0. \quad (4)$$

Here, $A = \langle x_i^2 \rangle M$, $\zeta = \sum_{i=1}^{M} x_i^2 - A$ and

$$\eta = \sum_{i=1}^{M} x_i \xi_{i+M} + b_l \sum_{i=1}^{M} x_i \xi_i + \sum_{i=1}^{M} \xi_i \xi_{i+M}.$$

For DCSK, $A = E_b/2$. We shall require that x_i is stationary, and that the correlations between x_i and x_{i+k} decay quickly as $|k|$ increases (which is standard for chaotic systems). We further assume that M is much larger than the characteristic correlation decay times. Under these assumptions, as M increases, the distributions of ζ and η approach Gaussian distributions [23]. Because the distributions of ζ and η are nearly Gaussian, it is sufficient to compute the variance of $\eta + \zeta$ to fully characterize the distribution of the interference. It is easy to check that because x_i is statistically independent from ξ_j for any (i, j), and ξ_i is statistically independent from ξ_j for any $i \neq j$, the cross-correlation of ζ and η, $\langle \eta\zeta \rangle$ is zero. Therefore, $\sigma_{\eta+\zeta}^2 = \sigma_\eta^2 + \sigma_\zeta^2$. Similarly, one can verify that the cross-correlations between the three terms in η are also zero, and therefore the variance of η is given by:

$$\begin{aligned} \sigma_\eta^2 =& 2\left\langle \sum_{i=1}^{M} x_i^2 \right\rangle \sigma_0^2 + \sigma_0^4 M \\ =& E_b \sigma_0^2 + \sigma_0^4 M \\ =& E_b \frac{N_0}{2} + \frac{N_0^2}{4} M. \end{aligned}$$

Here, we introduce $N_0 = 2\sigma_0^2$, which for the case of continuous signals has the meaning of the channel noise spectral power density. The variance of ζ depends on a specific chaotic system generating chaotic spreading sequences. In this paper, we shall use for this purpose the *symmetric tent map*:

$$x_{i+1} = 1 - D|x_i|, \qquad D = 2.$$

For this map, the distribution of x_i is uniform and the variance of ζ can be easily computed

$$\sigma_\zeta^2 = \frac{E_b^2}{5M}.$$

Thus, the contamination $\zeta + \eta$ is a random Gaussian variable with zero mean and variance

$$\sigma^2 = E_b \frac{N_0}{2} + \frac{E_b^2}{5M} + \frac{N_0^2}{4} M. \quad (5)$$

Because $\zeta + \eta$ is zero-mean Gaussian

$$\mathrm{BER} = P((\zeta + \eta) > A) = \frac{1}{2}\mathrm{erfc}\left(\frac{A}{\sqrt{2}\sigma}\right), \quad (6)$$

where $\mathrm{erfc}(x)$ is the complimentary error function

$$\mathrm{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_x^\infty \exp(-t^2)\, dt.$$

Thus, the bit error rate (BER) for the DCSK is given by

$$\mathrm{BER} = \frac{1}{2}\mathrm{erfc}\left(\sqrt{\frac{E_b}{4N_0}\left(1 + \frac{2}{5M}\frac{E_b}{N_0} + \frac{N_0}{2E_b} M\right)^{-1}}\right). \quad (7)$$

The term reflecting the variability of $\sum_{i=1}^{M} x_i^2$ for chaotic processes is the main difference between the BER expressions for DCSK and for periodic TR systems [18]. In Fig. 4(a), we present the results of numerical simulations with different values of M. Channel noise ξ_i was taken to be Gaussian. The bit error rate for conventional binary phase-shift keying (BPSK) $BER_{\mathrm{BPSK}} = \mathrm{erfc}(\sqrt{E_b/N_0})/2$ is also shown for comparison. In Fig. 5, we observe excellent agreement between the analytical prediction and the results of numerical simulations for $M = 100$.

From Fig. 4(a), we also see that at large M the performance degrades with increasing M, which is consistent with (7). This trend occurs due to the increasing contribution of noise-noise cross terms $\xi_i \xi_{i-M}$ in (1) and is typical for the correlation decoding of TR signals. As we increase M, keeping E_b/N_0 constant at a fixed signal amplitude, we increase N_0 proportionally to M. Thus, while the useful signal in (1) increases linearly with M, and so does the standard deviation of $\sum_{i=1}^{M} x_i(b_l \xi_{i-M} + \xi_i)$, $\sim\sqrt{MN_0} \sim M$, the standard deviation of $\sum_{i=1}^{M} \xi_i \xi_{i-M}$, $\sim\sqrt{MN_0^2} \sim M^{3/2}$, grows faster.

B. CDSK Performance

The correlator output can again be written in the form of (4) with $A = E_b/2$ and $\eta = \sum_{i=1}^{M} \eta_i$, where η_i is given by (3). Assuming $L \geq M$, variance of η is given by

$$\sigma_\eta^2 = E_b N_0 + \frac{3E_b^2}{4M} + \frac{N_0^2}{4} M.$$

The variance of ζ for the tent map used as a chaos generator is the same as in the case of DCSK. Thus, for the variance of the interference we have

$$\sigma^2 = E_b N_0 + \frac{19E_b^2}{20M} + \frac{N_0^2}{4} M \quad (8)$$

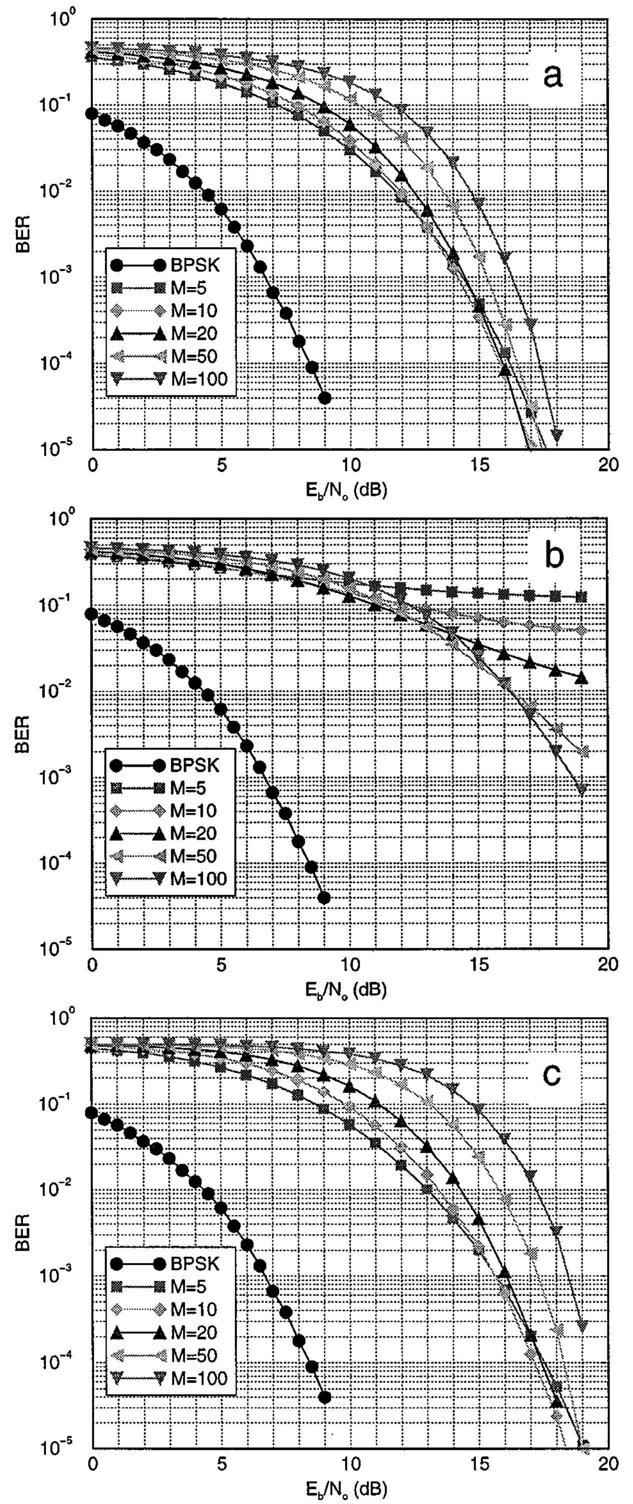

Fig. 4. Performance of correlation-based detection methods: DCSK (a), CDSK (b) and SCSK (c).

Thus, the bit error rate is given by

$$\mathrm{BER} = \frac{1}{2}\mathrm{erfc}\left(\sqrt{\frac{E_b}{8N_0}\left(1+\frac{19}{20M}\,\frac{E_b}{N_0}+\frac{M}{4}\,\frac{N_0}{E_b}\right)^{-1}}\right). \tag{9}$$

The results of numerical simulations with $L = 200$ are shown in Fig. 4(b). The comparison between the analytical and numerical results is given in Fig. 5. Considering that x_i and x_j can be considered statistically independent only approximately at large M, the analytical and the simulation curves at $M = 100$ match reasonably well. In Fig. 5, we also see the CDSK performs 2–3 dB worse than the DCSK. This is due to two factors. First, due to the nature of the transmitted signal, there are four signal–noise cross terms in (3), compared to only two such terms in (1) for DCSK. Second, in addition to interference terms due to noise (noise–signal and noise–noise terms), there are three interference terms due to noncomplete orthogonality of chaotic segments on two consecutive time intervals. Since these terms are present even when noise amplitude is zero, bit error rate saturates at large E_b/N_0 at the value $\mathrm{BER}_{\mathrm{sat}} = \mathrm{erfc}(\sqrt{5M/38})/2$. This saturation is visible in Fig. 4(b), which shows the bit error rate curves computed numerically for different values of M. Here, we also see that, as in the case of DCSK, increasing M at constant E_b/N_0 leads to performance degradation.

C. SCSK Performance

In general, the correlator output (3) for this system can be written in the form (4) with $A = E_b + M\Delta E_b$, where $\Delta E_b = \langle (F(x_{i-1} + \tilde{\xi}_{i-1}) - F(x_{i-1})F(x_{i-1})\rangle$. η and ζ can be defined as in the previous two cases. When M is large, ζ is a zero mean Gaussian variable with the variance in the case of the tent map $\sigma_\zeta^2 = 4E_b^2/(5M)$. η for SCSK is defined as

$$\eta = \sum_{i=1}^{M} F\left(x_{i-1} + \tilde{\xi}_{i-1}\right)\tilde{\xi}_i + \sum_{i=1}^{M}\left(F\left(x_{i-1} + \tilde{\xi}_{i-1}\right) - F(x_{i-1}))F(x_{i-1}) - \Delta E_b\right)$$

and in the limit of large M is also a zero-mean Gaussian variable. Because of the nonlinearity of F, it is difficult to find analytically the explicit formula for the bit error rate, however, we may expect to see the same general features in its performance as in DCSK and CDSK. Fig. 4(c) shows the numerical performance curves for SCSK. This figure confirms our expectation that the performance of SCSK should follow the same trends as that of DCSK or CDSK. In particular, we again observe the degradation of performance at large values of M.

D. Dependence of the Performance on the Spreading Sequence Length

Increasing the spreading sequence length degrades the performance of all three schemes only when the spreading sequences are rather long. A closer inspection of Fig. 4 reveals that sometimes as M increases with E_b/N_0 held constant, the performance first improves somewhat before it starts declining. This is even more obvious from Fig. 6, where bit error rates are presented as functions of the spreading sequence length M for several values of E_b/N_0.

Although for small M the expressions for bit error rates derived in the previous section are not exactly valid, we see that they nonetheless explain the existence of the optimal spreading sequence length. As a matter of fact, for CDSK the analytical

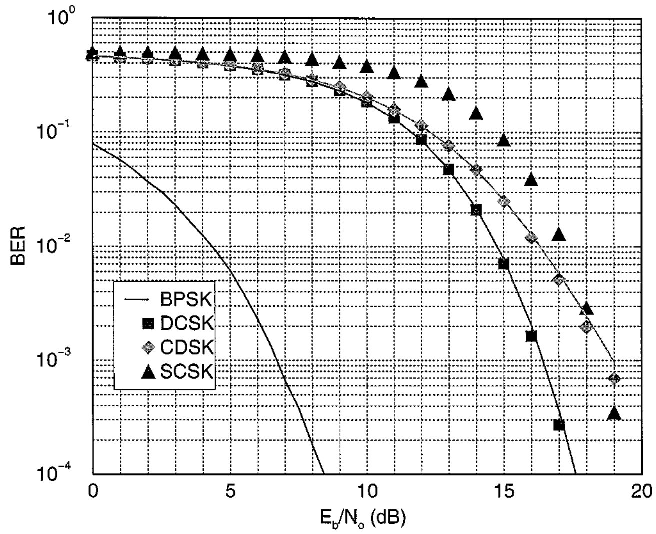

Fig. 5. Performance of DCSK, CDSK, and SCSK with $M = 100$. Numerical data and analytical estimates are shown with symbols and lines respectively.

expression accurately describes the performance of the system even at quite small values of M. This is because BER behavior at small M is primarily dictated by the first three η-terms in (3) and not by the ζ-term. The η-terms have symmetric probability distribution even for small M, while the ζ-term at small M has strongly nonsymmetric distribution, due to the restriction $\sum_{i=1}^{M} x_i^2 > 0$. For this reason, our theory can only qualitatively describe the dependence of DCSK BER at small and moderate values of M, while the description of CDSK is more precise.

For all three schemes, there are intermediate values of $M \sim 10 \ldots 100$ that minimize bit error rates at fixed E_b/N_0. Observing the good agreement of the analytical and numerical results for the CDSK, we can analytically estimate optimal performance of the correlation-based detector. Minimizing the bit error rate given by (10), we find that the error is minimum at $M = \sqrt{(19/5)}E_b/N_0$. The corresponding optimal performance is given by

$$BER_{\mathrm{CDSK\,opt}} = \frac{1}{2}\mathrm{erfc}\left(\sqrt{\frac{E_b}{8N_0}\,\frac{1}{1+\sqrt{\frac{19}{20}}}}\right)$$

which is ~12 dB worse than BPSK performance.

It should be kept in mind, however, that in many applications it may be more appropriate to characterize the performance of a communication system by the dependence of the bit error rate on the channel SNR $= P_s/P_n$ (where P_s is the signal power and P_n is noise power). Since $E_b/N_0 = 2P_sM/P_n = 2\mathrm{SNR}M$, increasing M will result in lower bit error rates at a fixed SNR for all three schemes considered here. In particular for DCSK and for CDSK, we can write

$$BER_{\mathrm{DCSK}} = \frac{1}{2}\mathrm{erfc}\left(\sqrt{\frac{1}{2}\,\mathrm{SNR}M\left(1+\frac{4}{5}\mathrm{SNR}+\frac{1}{4\mathrm{SNR}}\right)^{-1}}\right)$$

$$BER_{\mathrm{CDSK}} = \frac{1}{2}\mathrm{erfc}\left(\sqrt{\frac{1}{4}\mathrm{SNR}M\left(1+\frac{19}{10}\mathrm{SNR}+\frac{1}{8\mathrm{SNR}}\right)^{-1}}\right).$$

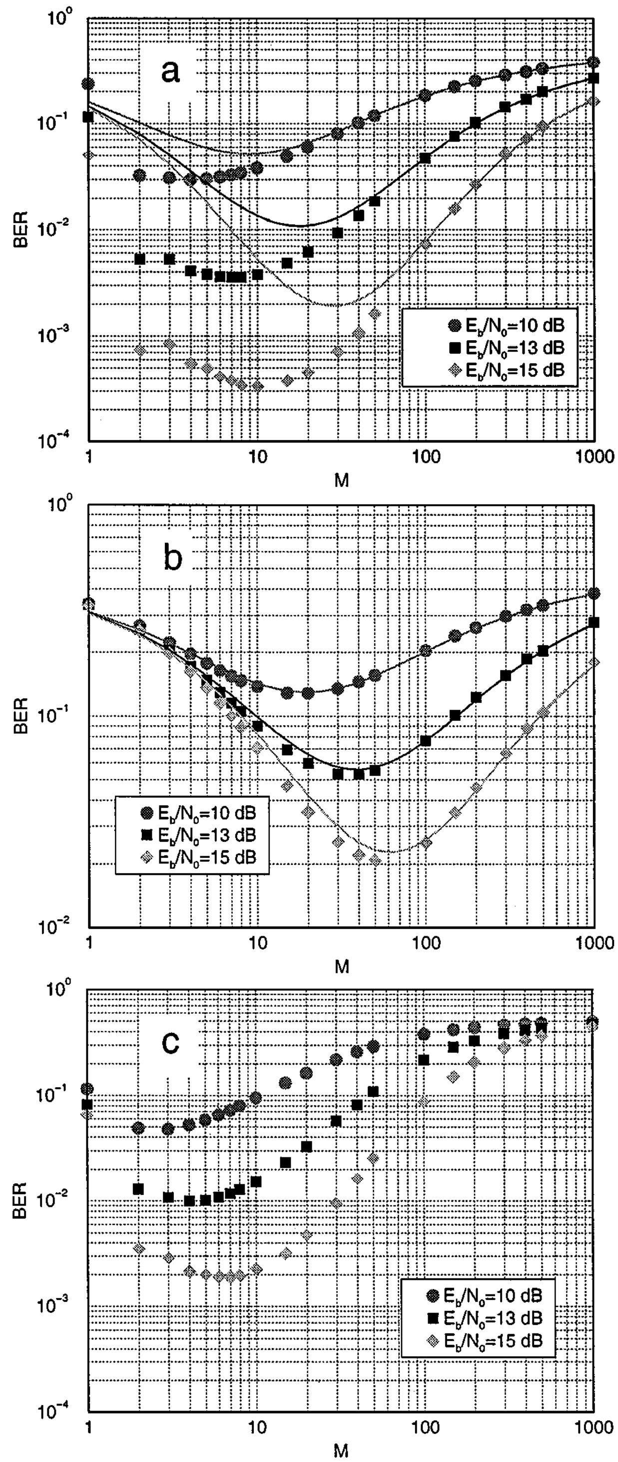

Fig. 6. Performance versus the spreading sequence length for DCSK (a), CDSK (b) and SCSK (c). Numerical data and analytical estimates are shown with symbols and lines respectively.

For SCSK, this dependence obtained by direct numerical simulation is shown in Fig. 7. We see that at large SNR the performance of SCSK follows approximately the same trend as BPSK and DCSK, namely, BER depends mainly on the product

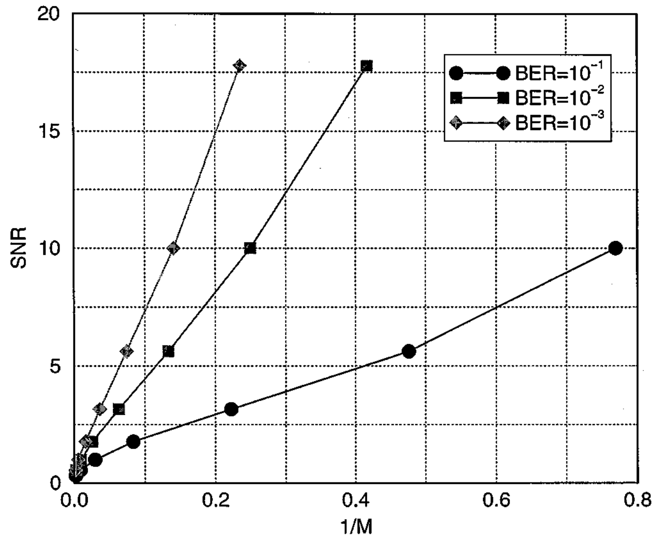

Fig. 7. SCSK performance: the dependence of SNR on M for constant BER's.

SNRM. Larger values of M allow to maintain the same level of communication reliability at lower levels of SNR: SNR $\sim 1/M$.

IV. Conclusion

In this paper, we analyzed and compared the performance of three base-band chaos-based spread-spectrum communication schemes. The first scheme, DCSK, was proposed earlier by Kolumbán *et al.* It involves the transmission of a segment of chaotic signal twice, first as a reference signal, and second, polarity-modulated by the information bit. The second scheme, CDSK, proposed here, is similar to DCSK, however the reference and the information-modulated signal are not transmitted sequentially, but added together with a predefined time delay. The third scheme, SCSK, is based on the specific choice of chaos generating system, which can recreate the reference signal at the receiver from the information-modulated chaotic signal.[2] The common feature of all three schemes is the correlation detection block providing a processing gain. Our BER analysis revealed that CDSK and SCSK perform 2–3 dB worse than DCSK. However, both CDSK and SCSK have simpler designs and the benefit of nonrepeating chaotic carrier, which yields the lower probability of detection. Additionally, for the proper operation of SCSK, the parameters of chaotic generators at the transmitter and the receiver have to be matched, which provides this scheme with enhanced selectivity. Both theoretically and numerically, we found an interesting feature of all three schemes, namely, that the BER at fixed E_b/N_0 reaches its minimum at a certain length of the spreading sequence M. This is not typical for standard communication techniques such as BPSK for which BER is independent on M at fixed E_b/N_0. This behavior is caused by the noise–noise cross-correlation at large M and fluctuations in the bit energy E_b at small M. The existence of the optimal M sets the base-band systems studied here aside from their narrow-band counterparts, FM-DCSK and PM-DCSK [18]–[20], where the energy per bit is constant and the dependence of BER on M is monotonic. Our calculations were performed using discrete-time chaotic systems for generation of spreading sequences for of all three schemes, but the results should also apply to their continuous-time implementations.

[2] See [20] for the description of a similar scheme for a narrow-band communication system.

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